Activity 15: Vectors from A to B

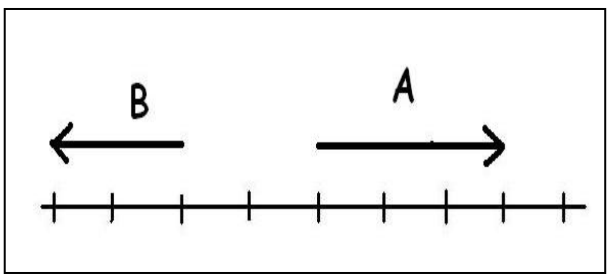
Teacher's Guide:

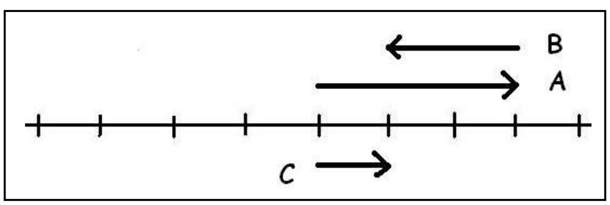
In order to understand the THEMIS Magnetometer line-plot data, students must first understand vectors. We introduce the concept of a vector, and point to additional web-based resources for teaching about vectors. Velocity is the most common and intuitively familiar form of a vector quantity, and we will start with this as an example.

Remind students that when they are in a car, there are two things that are the most important about the 'experience' : How fast are they going, and in what direction. We call this motion a vector because it consists of both a magnitude and a direction. One of these features, by itself, is not enough to completely describe how a car is moving at a particular moment.

Because a vector also requires a direction to specify it, it requires some kind of reference basis or 'coordinate system'. The simplest coordinate system for 1-dimensional vectors is the number line. Let's see how this works.

Vectors in 1-dimension.



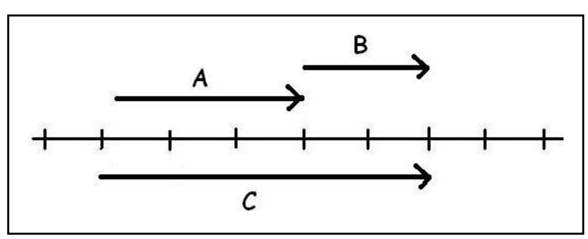


Above Figure: To add two vectors, A + B, place the tail of the arrow for B at the head of the arrow for A. The result is a third vector C which is called the Resultant. Note that the magnitude of C is 3 - 2 = 1 unit.

Right Figure: To subtract vectors, A - B, reverse the direction of B, and place it at the head of A. Note that the magnitude of C is 3 + 2 = 5 units.

Define two vectors called **A** and **B**. The **A** vector says 'Move three units to the right'. The **B** vector says 'Move 2 units to the left'. They look like the figure to the left. The length (magnitude) of Vector A is '3 Units' and its direction is 'Right'.

Suppose we added Vector **B** to Vector **A.** This could represent a person walking three blocks north on a street, then turning around and walking two blocks south on the same street. Although the total distance traveled is 3 + 2 = 5 blocks, this really doesn't tell us where the traveler ended up. To find this, we have to include in the addition the direction information at the same time.

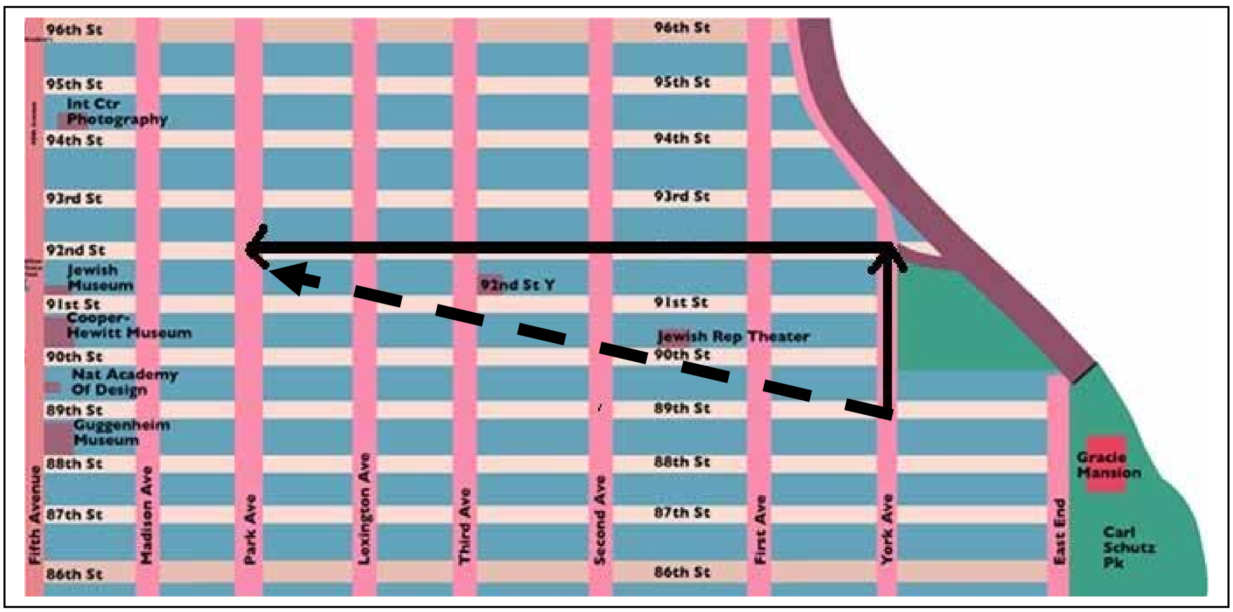


Vectors in 2-dimensions.

The previous example was simple, and can be used in systems that describe motion along a line, like water flowing down a hose, or relay sprinters on a 100-meter straight track. There are MANY of these kinds of problems. Can you and your students come up with other examples of 1-dimensional motion?

Motion in 2-dimensions is just a little more complicated. Think of the motion of balls on a pool table, or ATVs driving across the Bonneville Salt Flats. If you forget about vertical direction, traveling by car on the surface of Earth is also 2-dimensional motion. Because objects move, they can also be described by velocity vectors that are 2-dimensional. Again, you have to specify a coordinate system to serve as a direction reference. **With an Earth globe, show students that cars moving on Earth's surface travel north and south along directions of latitude, and east to west along directions of longitude.** This provides a simple 'geographic' coordinate system that we also use in city driving…especially now that many people have GPS systems. You can also use an ordinary compass to get the same coordinate 'bearings'.

In our previous example, lets assume that our New York City shopper traveled 3 blocks north on York Avenue along vector **A**, but then traveled 5 blocks west on 92nd Street on vector **B**. Let's see what the shopper's path looked like in the figure below.



Adding the two vectors, **A+B**, we see that, although the total distance walked is 3 + 5 = 8 blocks, the total distance from where the shopper started is a different length, and is represented by the dotted vector **C** in the above figure. Because the streets are perpendicular, the triangle formed by vectors **A**, **B** and **C** is a right-triangle, and **C** is the hypotenuse. This means that the total distance from the starting point is given by the sums of the squares of the magnitudes of the vectors **A** and **B** so that $C^2 = A^2 + B^2$ so that the magnitude of vector **C** is just the square-root of (25 + 9) which is 5.8 blocks. This would be called the 'as the crow flies' distance.

What does this have to do with velocity?

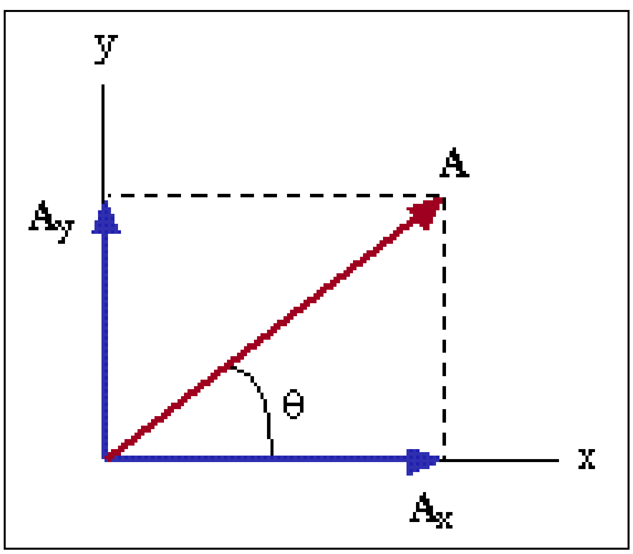
The above example described a vector quantity called 'position', but we can just as easily use this same set up to describe velocity. Imagine that a crow is actually flying along the position vector **C** that connects the shopper's starting position at the corner of 89th Street and York Avenue, with the shoppers destination at the corner of 92nd Street and Park Avenue. Let's call the crow's velocity vector **V**. Now the question is, how fast is the bird moving in a direction along York Avenue, and along 92nd Street?

You can see from the street map that, as the crow moves along the diagonal, its 'shadow' will travel at a certain speed along each of these two streets. This means that the vector **V** for the crow, can be thought of as two other vectors, call them **Vn** and **Vw,** added together. If we take the **Vn** vector that runs along York Avenue, and add to its head, the **Vw** vector that runs along 92nd Street, vector **V** is the Resultant vector. We can write this as the vector addition equation:

$$\mathbf{V} = \mathbf{Vw} + \mathbf{Vn}$$

Vector components:

A 2-dimensional vector is completely defined by the sum of the components of the vector along two coordinate axis. For example, let's look at the ordinary Cartesian plane with axis X and Y in the figure below.



The vector A is 'resolved' into two vectors Ax and Ay. Another way to look at these component vectors is that

Ax = |Ax| **x**
Ay = |Ay| **y**

Where |Ax| and |Ay| are the **magnitudes** of these vectors, and **x** and **y** are the direction vectors along the two axis for which |x| = |y| = 1.

Another important thing to see from this Cartesian coordinate system is that, with a little bit of trigonometry:

$$|Ax| = |A| \cos(\theta) \quad \text{and} \quad |Ay| = |A| \sin(\theta)$$

The nice thing about working with vector components is that you can now add and subtract vectors very easily. For example consider two vectors

$$\mathbf{A} = |Ax|\,\mathbf{x} + |Ay|\,\mathbf{y}$$
$$\mathbf{B} = |Bx|\,\mathbf{x} + |By|\,\mathbf{y}$$

Then to add them to get the vector **C** you have

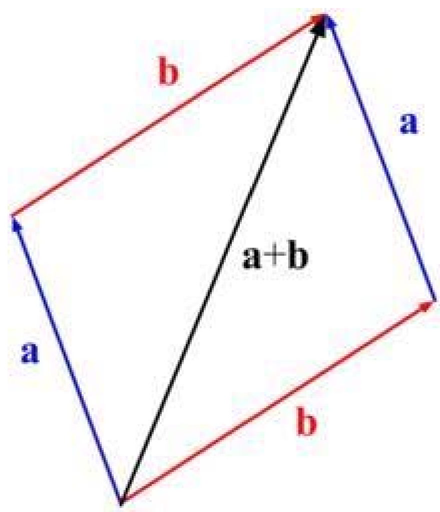
$$\mathbf{C} = (|Ax| + |Bx|)\,\mathbf{x} + (|Ay| + |By|)\,\mathbf{y}$$

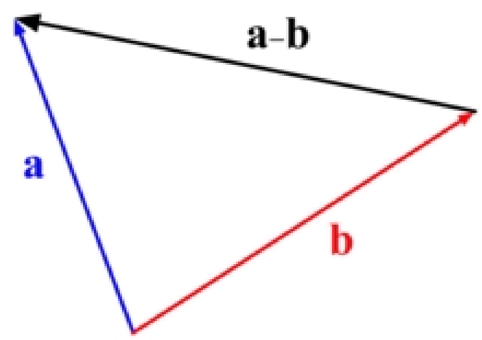
Similarly, to subtract them you have

$$\mathbf{C} = (|Ax| - |Bx|)\,\mathbf{x} + (|Ay| - |By|)\,\mathbf{y}$$

If you prefer adding vectors graphically, draw vector A on the Cartesian plane, and then draw vector B starting at the head of vector A to create the vector **A+B** as in the figure below to the left.

If you want to subtract these two vectors, draw vector A, then draw vector B starting at the base of vector A, draw a vector connecting the two tips of A and B to find **A-B** as in the figure below right.





Additional Resources

http://exploration.grc.nasa.gov/education/rocket/vectors.html

Procedure

Give the students a lecture about how to add and subtract vectors. Have the students who understand how it works solve a couple examples of vector addition and subtraction problems in front of the other students. Have the students answer the student worksheet to assess if they can add and subtract vectors in 2-dimensions. This is important before introducing the three dimensional magnetometer data.

Student's Name ___________________________ Date _______

Problem 1) A car travels along a path such that its speed is 30 miles per hour north and 25 miles per hour west. What is the total speed of the car along its actual path?

Problem 2) A jet plane takes off from O'Hare International Airport in Chicago. It is headed in a direction due West with a speed of 550 miles per hour. There is a wind blowing from the south to the north at a speed of 150 miles per hour.

A) Use vector addition to diagram the two vectors and calculate the resultant vector, which is the jets speed relative to the ground.

B) What is the direction of the jet's velocity vector relative to the ground?

Problem 3) On a piece of paper, iron filings are sprinkled to reveal the magnetic field of a bar magnet. At a particular point on the paper, the magnetic field vector is given by:

$$\mathbf{B1} = -15 \text{ gauss } \mathbf{X} + 10 \text{ gauss } \mathbf{Y}$$

On a second piece of paper, the iron filings from a second magnet are revealed using iron filings. At the same point on the paper as for the first magnet, a measurement is made of the magnetic field vector and it is given by

$$\mathbf{B2} = 26 \text{ gauss } \mathbf{X} - 5 \text{ gauss } \mathbf{Y}$$

A) If both magnets were placed under a third piece of paper at the same location, and iron filings were sprinkled on the paper, what would be net sum of the two magnetic fields at the point used in the first two papers?

B) What would be the difference in magnetic field strengths between the two magnets at the measurement point?

C) Which bar magnet has the strongest magnetic field?

Answer Sheet

Problem 1) A car travels along a path such that its speed is 30 miles per hour north and 25 miles per hour west. What is the total speed of the car along its actual path?

Answer: speed = square-root (30^2 + 25^2) = 39 miles per hour.

Problem 2) A jet plane takes off from O'Hare International Airport in Chicago. It is headed in a direction due West with a speed of 550 miles per hour. There is a wind blowing from the south to the north at a speed of 150 miles per hour.

A) Use vector addition to diagram the two vectors and calculate the resultant vector, which is the jets speed relative to the ground.

Answer: speed = square-root (550^2 + 150^2) = 570 miles per hour.

B) What is the direction of the jet's velocity vector relative to the ground?

Answer: Northwest. For 'experts' the angle is arcTan (150/550) = 15 degrees north of west.

Problem 3) On a piece of paper, iron filings are sprinkled to reveal the magnetic field of a bar magnet. At a particular point on the paper, the magnetic field vector is given by:

B1 = -15 gauss **X** + 10 gauss **Y**

On a second piece of paper, the iron filings from a second magnet are revealed using iron filings. At the same point on the paper as for the first magnet, a measurement is made of the magnetic field vector and it is given by

B2 = 26 gauss **X** - 5 gauss **Y**

A) If both magnets were placed under a third piece of paper at the same location, and iron filings were sprinkled on the paper, what would be net sum of the two magnetic fields at the point used in the first two papers?

Answer: **B1 + B2** = (-15 + 26) **X** + (10 -5) **Y** = 11 gauss **X** + 5 gauss **Y**

B) What would be the difference in magnetic field strengths between the two magnets at the measurement point?

Answer: B1 - B2 = (-15 -26) **X** +(10 +5) **Y** = -41 gauss **X** + 15 gauss **Y**

C) Which bar magnet has the strongest magnetic field?
Answer: Find the magnitude of B1 and B2 and compare.

|B1| = square-root ($(-15)^2$ + $(10)^2$) = 18.0 gauss.
|B2| = square-root ($(26)^2$ + $(-5)^2$) = 26.5 gauss. **This is the strongest.**