Homework Set 7

Due Date Tuesday, November 23, 1999, at the beginning of class.

1 Cosmological Distances

Part A Cepheid Variables

As we've discussed in class there exists a relationship between the period of luminosity variation and average luminosity for pulsating stars known as Cepheids. If we can identify a Cepheid variable and measure the period of its brightness variation then we can infer its luminosity from this relation and in turn determine its distance. We accomplish this via the relation between brightness and luminosity given as

$$b = \frac{L}{4\pi d^2}$$

where b is brightness measured in $\mathrm{Watts\,m^{-2}}$, L is luminosity typically measured in $\mathrm{L_\odot}$ where $1\mathrm{L_\odot} = 3.826 \times 10^{26}$ Watts, and d is distance to the star measured in parsecs, with $1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m}$.

1. The Hubble Space Telescope (HST) Key Project team has been searching for Cepheids in galaxies within the Virgo cluster. In the barred spiral galaxy NGC 4535, believed to be a Virgo cluster member, several Cephieds have been found. One of them has a period of 30 days and is measured to have a brightness at Earth, $b = 1.25 \times 10^{-18}\,\mathrm{Watts\,m^{-2}}$. Use the period-luminosity relationship given in Figure 1 and calculate the distance to NGC 4535 in Mpc.

2. Consider a Cepheid with period of 100 days. If the limiting brightness that HST can see is $b = 6.44 \times 10^{-19}\,\mathrm{Watts\,m^{-2}}$ calculate the distance to which this Cepheid can be seen. In practice the limiting distance for Cepheids is right about at the distance of the Virgo cluster. What might be some reasons for this limitation?

Part B Light Echoes

In 1987 a supernova exploded in the Large Magellanic Cloud, it was given the endearing name SN 1987A. When the Hubble Space Telescope (HST) observed the remanant of the supernova it observed several rings surrounding it (see figure 2). These are thought to be material thrown off by the progenitor star sometime before the explosion. The supernova heated the rings of gas causing them to glow like fiery goats. Let's consider the inner, bright ring. It is presumed to be circular but tilted from the perspective of Earth. It's projected semi-axes have dimensions 0.830" and 0.605". Because of the tilted geometry light was was received from the near side of the ring 340 days before light arrived from its far side.

1. Determine the inclination angle, i, between the normal to the plane of the ring and the line of sight from Earth.

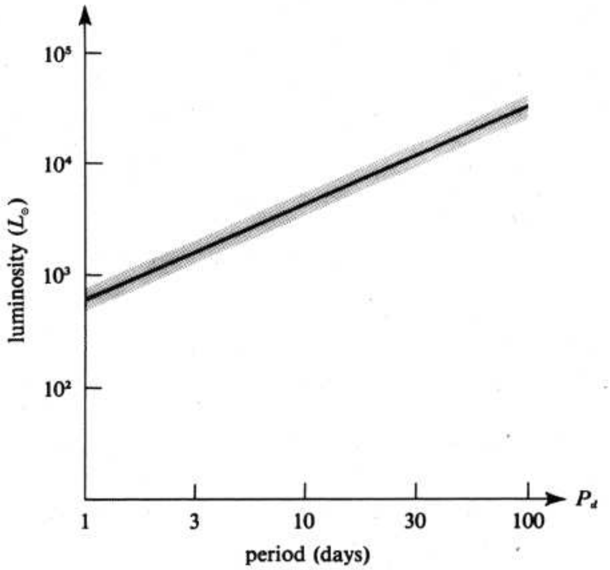

Figure 1: The Period-Luminosity relationship for Cepheids. The shaded strip centered on the mean line indicates schematically that there are slight devaitions from a one-to-one relation, because of both theoretical reasons and observational uncertainties. (From Shu, Chapter 9)

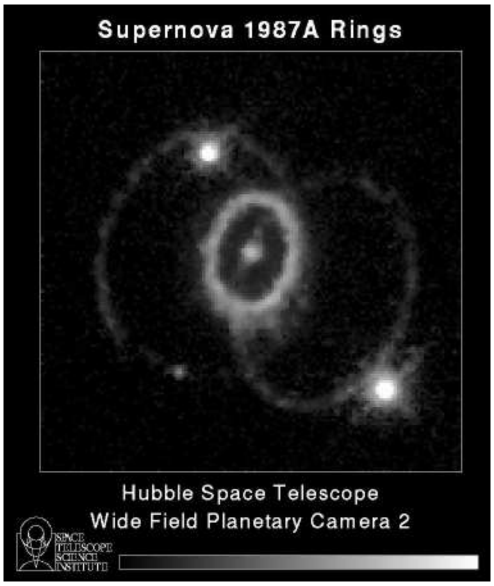

Figure 2:

2. Use the time delay to determine the radius, r, of the ring in parsecs.

3. Use trigonometry to find the distance to SN 1987A.

4. If HST can resolve a minimum angle of 0.100" what is the maximum distance to

which this method could be applied?

2 The Mass of the Local Supercluster

As we've discussed in class, there are deviations from the Hubble flow of galaxies known as *peculiar velocities*. At late times in the Universe's history these peculiar motions are due purely to gravitaional accelerations from asymmetric mass distributions on relatively small scales. The Local Group of galaxies sits at the edge of what is known as the Local Supercluster (see Figure 3) which is dominated by the Virgo cluster. The Local Group is being gravitationally pulled toward the Virgo cluster, this is called the **Virgocentric infall**.

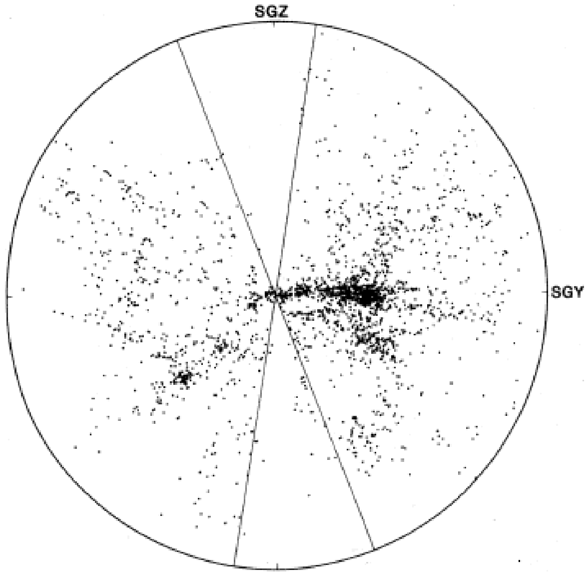

Figure 3: The distribution of 2175 bright galaxies, out to roughly 50 Mpc. The Local Supercluster extends to the right, with the Milky Way (at center) located near the edge of the supercluster. The plane of the Milky Way bisects the "empty" slices; galaxies within the slices are hidden from view by Galactic dust and gas (the zone of avoidance). (Figure from Tully, *Ap. J., 257,* 389, 1982.)

1. The giant elliptical galaxy M87 sits at the center of the Virgo cluster and is a good galaxy to use to estimate the value of the Virgocentric infall velocity (it should have no line of site peculiar velocity due to virial motions within the cluster). Its redshift is measured to be $z = 0.00314$ in the frame of the Local Group. Calculate its peculiar velocity, $v_{\rm r}$, assuming that the distance to the Virgo cluster is 16 Mpc, and $H_o = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

2. Now, we can write the infall velocity in terms of the gravitational acceleration

$$\mathrm{v(r)} = \mathrm{g(r)}t.$$

What time would we want to use? Write $v_{\rm r}$ in terms of G, the total mass of the Local Supercluster, M, the distance to the cluster, d, and your expression for time.

3. Doing this analysis in all its hairy-backed-Kansas-mountain-goat-ugliness would yield the expression
$$\mathrm{v(r)} = \frac{2}{3}\frac{1}{\Omega^{0.4} H_o}\mathrm{g(r)}$$
where Ω is the cosmological density parameter. After completing Problem #3 of this assignment come back to this part and tell me why this expression kinda makes sense. Use $\Omega = 1$ and the previous values for H_o, and distance, and also include your previous expression for g. Calculate the mass of the Local Supercluster (in solar masses) assuming that our motion relative to the cluster is not strongly disturbed by the tidal fields of more distant mass concentrations.

4. The total luminosity of the Local Supercluster is $L = 3 \times 10^{12}\,\mathrm{L}_\odot$. Compute the mass-to-light ratio (in units of $\mathrm{M}_\odot/\mathrm{L}_\odot$) and explain the result.

3 Physical Cosmology

In class we learned that for a critical Universe (i.e. one with exactly E = 0)
$$\dot{R}^2 = \frac{8\pi G\rho(t)}{3}R^2 \tag{1}$$
where R(t) is a scale factor indicative of the size of the Universe at time t.

1. How does $\rho(t)$ scale with R(t)? If the current density, i.e. ρ(today), is denoted ρ_o, and the current scale factor is denoted R_o, write $\rho(t)$ as a function of ρ_o, R_o, and R(t).

2. Plug this result into (1). Rearrange the equation into the form
(function of R)dR = (function of t)dt
and integrate to get R(t).

3. Use Hubble's law and the result of part 2 to find the age of the Universe, t_o, in terms of H_o. Let $H_o = 65\,\mathrm{km\,s^{-1}\,Mpc^{1}}$. Express t_o in years.

Now (1) is valid only for a critical E = 0 Universe. A more general calculation, allowing for nonzero energy, would show
$$\dot{R}^2 + kc^2 = \frac{8\pi G\rho(t)}{3}R^2 \tag{2}$$
where
$$\begin{array}{llll} k = 0 & \Rightarrow & E = 0 & \text{(Critical Universe)} \\ k = 1 & \Rightarrow & E < 0 & \text{(Bound Universe)} \\ k = -1 & \Rightarrow & E > 0 & \text{(Unbound Universe)} \end{array}$$

4. Now consider a universe with *no* matter. Evaluate (2) to find R(t).

5. What is the age of the Universe, t_o, as a function of H_o for this empty model? What is t_o in years if $H_o = 65\,\mathrm{km\,s^{-1}\,Mpc^{1}}$?

6. Given that the real Universe does contain goats and other matter, what can you say about the real age of the Universe relative to what you found in part 5?

We can make (2) even more general by including a term with Einstein's cosmological constant:

$$\dot{R}^2 + kc^2 = \frac{8\pi G\rho(t)}{3}R^2 + \frac{\Lambda c^2}{3}R^2. \tag{3}$$

7. Plug your answer for part 1 into (3). take the derivative with respect to time of both sides of the equation to show

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho}{3} + \frac{\Lambda c^2}{3} \tag{4}$$

8. Use the defininitions of the acceleration parameter q_o, the matter density parameter Ω_{M}, and the vacuum energy density parameter Ω_Λ from your notes (you know, the ones that you took in class) with equation (4) to find q_o as a function of Ω_{M} and Ω_Λ.

There is considerable observational evidence and theoretical research pointing toward the notion that we live in a critical k = 0 Universe.

9. If this is true, what is Ω_{total}? Express Ω_{M} in terms of Ω_Λ.

10. Let's say that some intrepid astronomer measures $q_o = -\frac{1}{2}$. What are the values of Ω_Λ and Ω_{M}? What are the implications to the expansion and fate of the Universe?