Homework Set 4

Due Date Thursday, October 7, 1999, at the beginning of class. Try to use the homework as part of your study for the **midterm** on **Tuesday, October 5, 1999**.

1 Spectroscopic Binary

Shu, Chapter 10, Problem 10.1.

2 Double-Neutron-Star Binary PSR B1913+16

The double-neutron-star binary discussed in lectures contains a neutron star from which we detect a pulsed radio signal, named PSR B1913+16, and a second neutron star which we cannot see. The radio pulses arise because PSR B1913+16 emits a beam of radio waves along its magnetic axis, which is tilted with respect to its rotation axis, and the beam sweeps past the Earth once per neutron-star rotation, as in Fig. 2.1. (Just like a lighthouse.) The second neutron star probably emits a beam of radio waves too, but this second beam presumably never sweeps past the Earth.

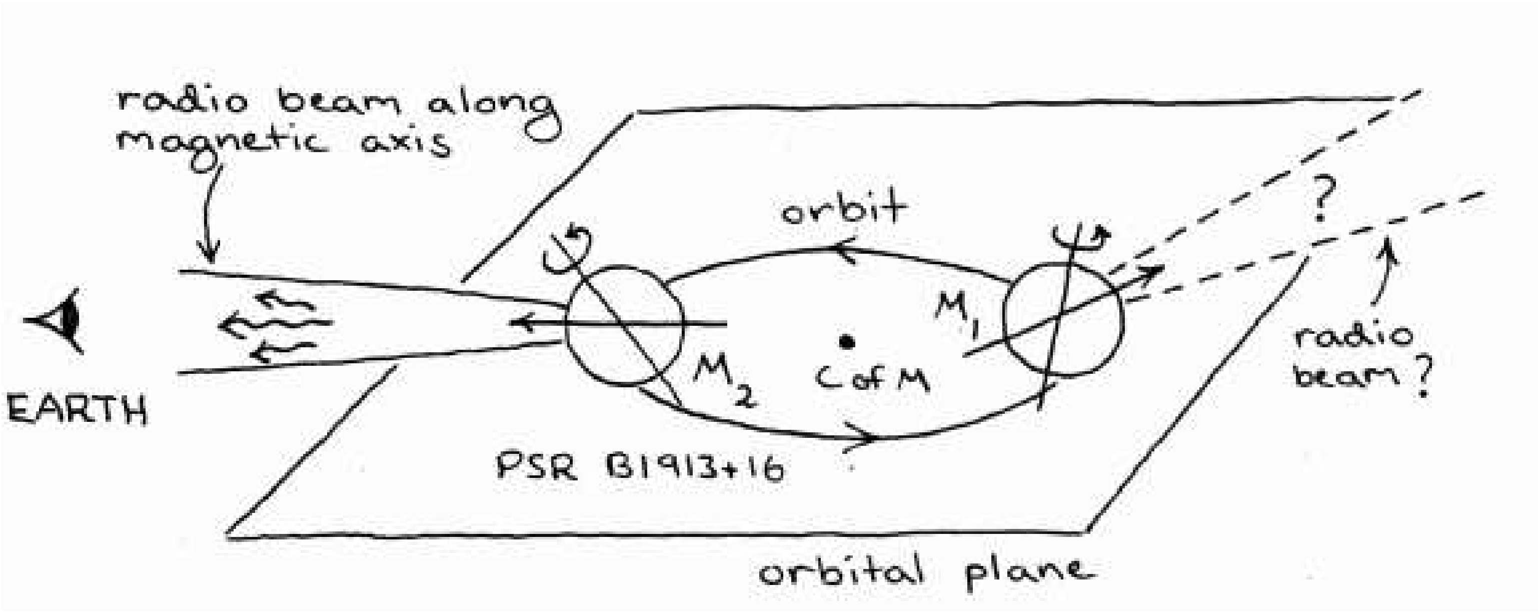

Figure 2.1:

PSR B1913+16 rotates once every $P_s = 59.0\,\mathrm{ms}$ as measured by an observer in its orbital rest frame. However, the separation of the radio pulses measured at Earth (i.e. the pulse period) fluctuates between $P_s - \Delta P_s$ and $P_s + \Delta P_s$, as shown in Fig. 2.2, mainly due to the **Doppler effect**. (This is related to, but not the same as, the Doppler shift of spectral lines discussed in lectures.)

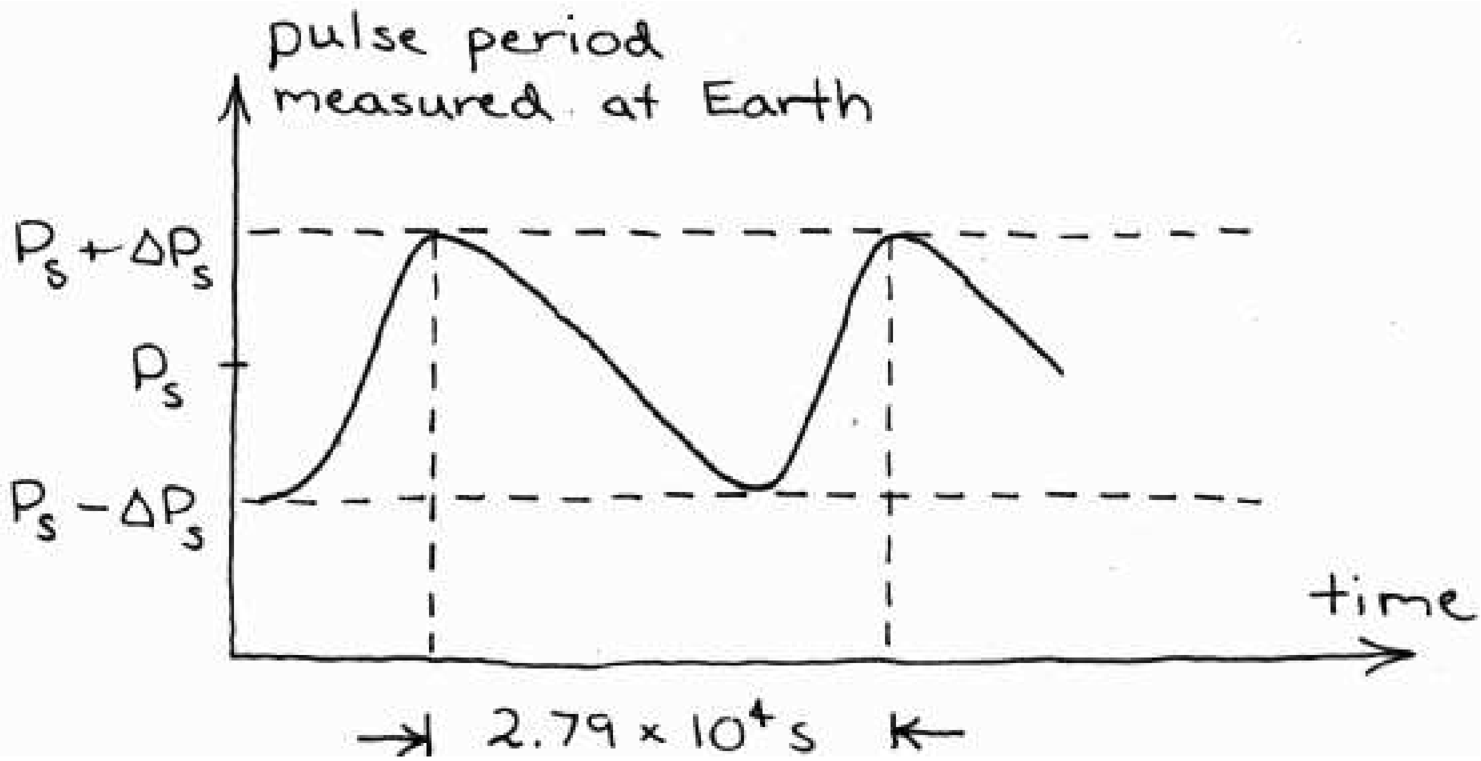

Figure 2.2:

1. Redraw Fig. 2.2 and mark three points where PSR B1913+16 is moving (i) towards Earth with maximum velocity along the line of sight, (ii) away from Earth with maximum velocity along the line of sight, and (iii) transverse to the line of sight.

2. What property of the binary system corresponds to the time interval of 2.79×10^4 s?

By studying the detailed shape of the curve in Fig. 2.2, including general relativistic effects, it is possible to deduce the eccentricity of the binary orbit, $e = 0.617$, and the masses of the two neutron stars, $M_1 = 1.39M_\odot$ and $M_2 = 1.44M_\odot$. (M_2 is PSR B1913+16.)

3. Using your answer to part 2. and the data in the preceding paragraph, calculate the lengths of the semimajor and semiminor axes of the orbit of M_1. Then do the same for M_2.

4. Using your answer to part 3., draw **approximately to scale** the two stellar orbits, placing the centre of mass at the origin. Choose your scale wisely so that the two orbits fit on a page and can be seen without a magnifying glass!

In reality, the pulse-period fluctuations ΔP_s in Fig. 2.2 are known to great precision; the pulse period is measured to more than ten decimal places (!) using an atomic clock. However, the preceding parts of the question contain enough information for us to calculate ΔP_s ourselves.

Firstly, we need to calculate the maximum velocity of PSR B1913+16 along our line of sight. Assume for simplicity, here and in what follows, that the orbital plane is inclined edge-on to the line of sight and the semimajor axis is transverse to the line of sight. (In reality, this is not the case; the inclination angle i is known to be $\sin i = 0.76 \pm 0.14$.) Note that we cannot just apply the results of Shu, Problem 10.1, because that problem pertains to circular orbits ($e = 0$).

5. It was shown in lectures that the relative position of the two stars, $\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1$, satisfies the 1-body Kepler equation of motion

$$\frac{d^2\mathbf{x}}{dt^2} = \frac{-G(M_1 + M_2)\mathbf{x}}{|\mathbf{x}|^3} . \tag{1}$$

Write down **without proof** the energy equation corresponding to (1). (Adapt eq. 3 on p. 23 of the lecture notes.)

6. By expressing E in terms of the semimajor axis a (adapt the result from p. 25 of the lecture notes), show that the energy equation corresponding to (1) takes the form

$$\left|\frac{d\mathbf{x}}{dt}\right|^2 = G(M_1 + M_2)\left(\frac{2}{|\mathbf{x}|} - \frac{1}{a}\right) . \tag{2}$$

7. Explain in words at what point in the orbit the velocity along the line of sight is a maximum. Substitute into (2) the value of $|\mathbf{x}|$ at this point and show that one has

$$\left|\frac{d\mathbf{x}}{dt}\right|^2_{\max} = \frac{G(M_1 + M_2)}{a}\frac{1+e}{1-e} . \tag{3}$$

8. The previous result is the maximum velocity of the **relative** position vector $\mathbf{x}$. Using the results presented in lectures, relate the **actual** position $\mathbf{x}_2$ of PSR B1913+16 to the relative position $\mathbf{x}$ (with the centre of mass at the origin). Hence find an expression for the maximum value of the velocity of M_2 (i.e. PSR B1913+16).

9. Evaluate numerically the maximum value of the velocity of PSR B1913+16. Express your answer in $\mathrm{m\,s^{-1}}$.

Now that we know the maximum velocity of PSR B1913+16, we relate it to the Doppler shift of the pulse period, ΔP_{s}.

Consider a flashlight blinking periodically, with period P_{s} in its rest frame. Suppose that, at $t = 0$, it blinks on at a distance L from the Earth, moving towards the Earth with speed V.

10. Find an expression, in terms of L and c, for the time t_1 when an observer on Earth sees the **first** flash from the flashlight. Find an expression, in terms of L, V, P_{s} and c, for the time t_2 when an observer on Earth sees the **second** flash from the flashlight. (A quick diagram will help you answer this question.)

11. Using the results of part 10., find an expression (in terms of V, P_s and c) for the period of the blinking flashlight measured at Earth.

12. Using the results of parts 9. and 11., evaluate ΔP_s in Fig. 2.2 numerically. Express your answer in ms.

3 Eclipsing Binaries

(**Quick question.**) Sketch (**roughly** to scale) schematic light curves of the following types of binaries. A light curve is a graph of total light intensity against time, similar to Fig. 10.5 in Shu.

1. Small, bright star A and large, dim star B. The diameter and brightness of A are one third and twice the diameter and brightness of B respectively. Both A and B are perfect spheres. Eclipses are total.

2. Two stars A and B of equal diameter and brightness. Both A and B have tidal bulges. Eclipses are total.

Suppose our Sun acquired a twin star of equal mass and diameter, forming a binary with a circular, 5-yr orbit. Let us call the system 'the Solar binary'.

3. What is the semimajor axis of the Solar binary?

4. Suppose Zog is an alien observer on some other star. In order for Zog to see the Solar binary as an **eclipsing binary**, what is the minimum inclination angle i of her line of sight with respect to the perpendicular of the orbital plane of the Solar Binary?

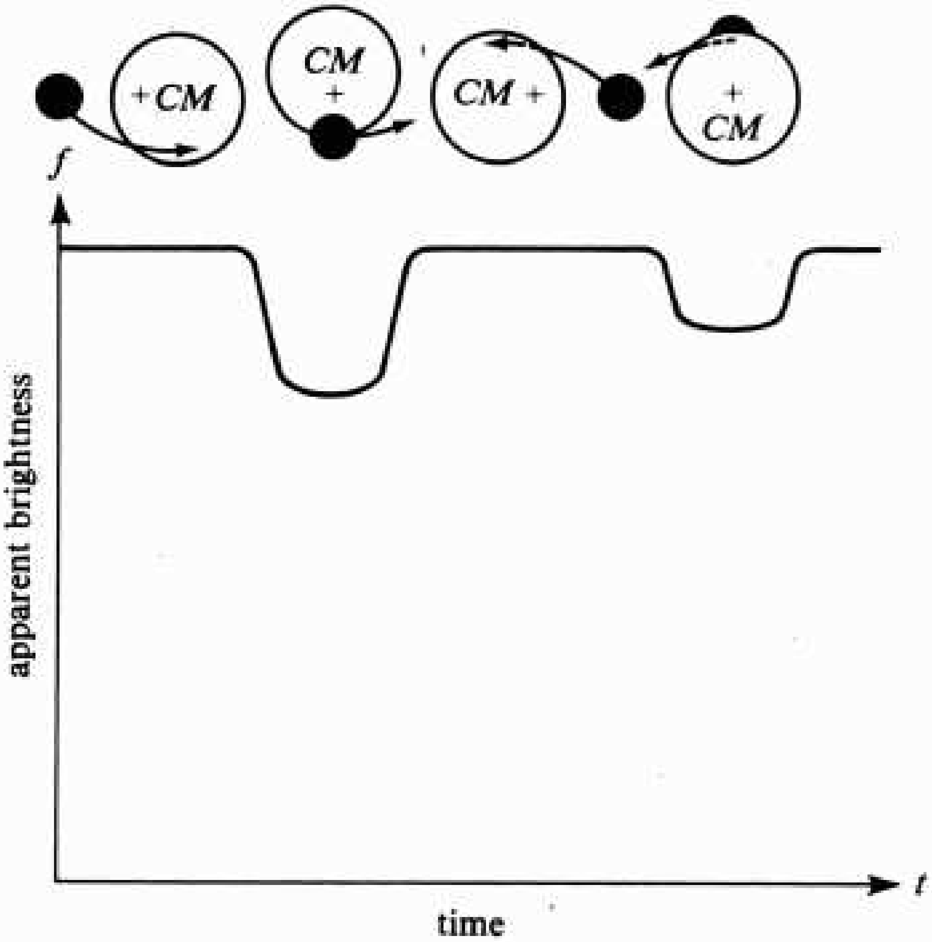

Figure 10.5. Schematic light curve of an eclipsing binary. Deeper eclipses result when the cooler star eclipses the hotter star than when the hotter star eclipses the cooler. The pattern of eclipses repeat once every binary period.