Homework Set 3

Due Date Thursday, September 23, 1999, at the beginning of class.

1 Formation of Saturn's Rings

It has been proposed that Saturn's rings formed when one or more small moons, originally orbiting outside Saturn's Roche limit, spiralled inwards over time due to tidal effects, crossed inside the Roche limit, and were disrupted.

1. Calculate the orbital angular frequency (in units of $\mathrm{rad\,s^{-1}}$) of a hypothetical moon travelling in a circular, prograde orbit at the Roche limit of Saturn. Compare your answer to Saturn's spin frequency (yes, we mean a number). If values for the needed quantities are not given in Shu's text then look in the appendices of the books listed as references in the syllabus.

2. Do tidal forces increase or decrease the orbital angular momentum of the hypothetical moon? Does the moon spiral in or out? Explain why using words and a diagram similar in style to the one on page 37 of the lecture notes.

3. Redo the previous question for a hypothetical moon in a **retrograde** orbit at the Roche limit of Saturn. Use a second diagram to illustrate your answer.

4. If the proposed scenario for ring formation is correct, in what sense (prograde or retrograde) do the rock fragments in the resulting rings revolve? Why?

One can measure the sense of revolution of Saturn's rings. It turns out that they revolve in the opposite sense to that predicted by the disruption scenario. Too bad for the scenario!

5. In a short paragraph, describe what you learnt from this problem, what you think the point of this problem is, and how it fits in to the material covered in lectures. (Please answer this seriously; it is a serious question, i.e. we will grade it!)

2 Child on a Swing

Consider a child of mass m sitting on a swing of length l, as in Figure 2.1. Suppose the child is being pushed and pulled by her big brother with a force $-F(t)\hat{\mathbf{x}}$.

1. What other forces are acting on the child?

2. Write down the x and y components of Newton's equation of motion for the child.

3. For small angles of swing, show that the x position of the child satisfies

$$m\frac{d^2x}{dt^2} = -\frac{mgx}{l} - F(t), \tag{1}$$

where g is the acceleration due to gravity at the Earth's surface.

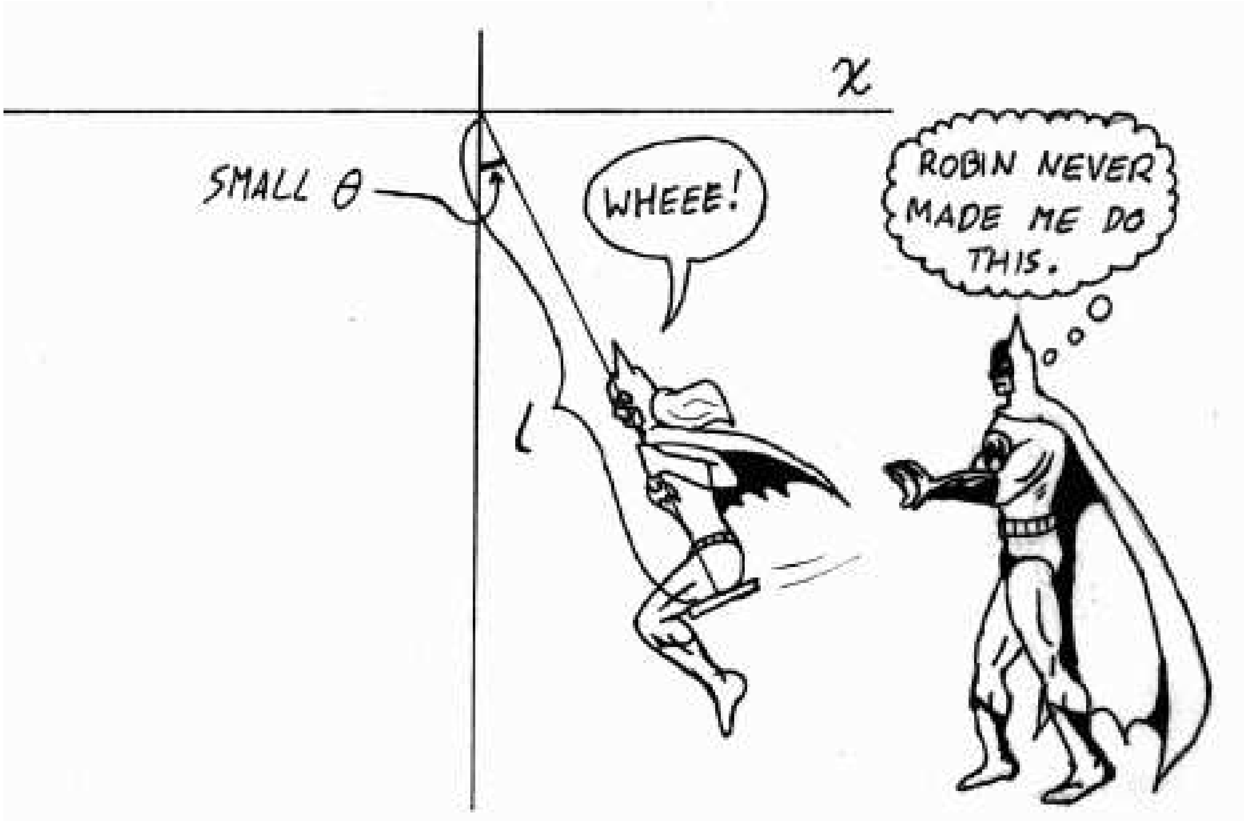


Figure 2.1:

Suppose that the brother pushes and pulls in a periodic fashion, with $F(t) = F_0 \cos \Omega t$. Also suppose that the initial position and velocity of the child are $x(0) = x_0$ and $\dot{x}(0) = 0$. The child's position as a function of time is then given by

$$x(t) = \left(x_0 + \frac{F_0/m}{\omega^2 - \Omega^2} \right) \cos \omega t - \frac{F_0/m}{\omega^2 - \Omega^2} \cos \Omega t \ . \tag{2}$$

(You do not need to prove this. However, if you wish to convince yourself that it is correct, just substitute $x(t)$ into (1) and check that the left-hand side equals the right-hand side, then check that it satisfies the stated initial conditions. An equation like (1) with a second derivative in it must always be accompanied by two initial conditions to specify a solution.)

4. Take $\omega = \Omega/2$ and $F_0/m\omega^2 = x_0$. Draw a sequence of diagrams at times $t = 0, \pi/4\omega, \pi/2\omega, 3\pi/4\omega, \pi/\omega, 5\pi/4\omega, 3\pi/2\omega, 7\pi/4\omega, 2\pi/\omega$ showing the successive positions of the swing and the direction and relative magnitude (as an arrow) of $F(t)$ at each position. (Draw the swing positions and arrows roughly to scale, but don't be too fussy.)

5. Redo the above question for $\omega = 2\Omega$ and times $t = 0, \pi/4\Omega, \pi/2\Omega, 3\pi/4\Omega, \pi/\Omega, 5\pi/4\Omega, 3\pi/2\Omega, 7\pi/4\Omega, 2\pi/\Omega$.

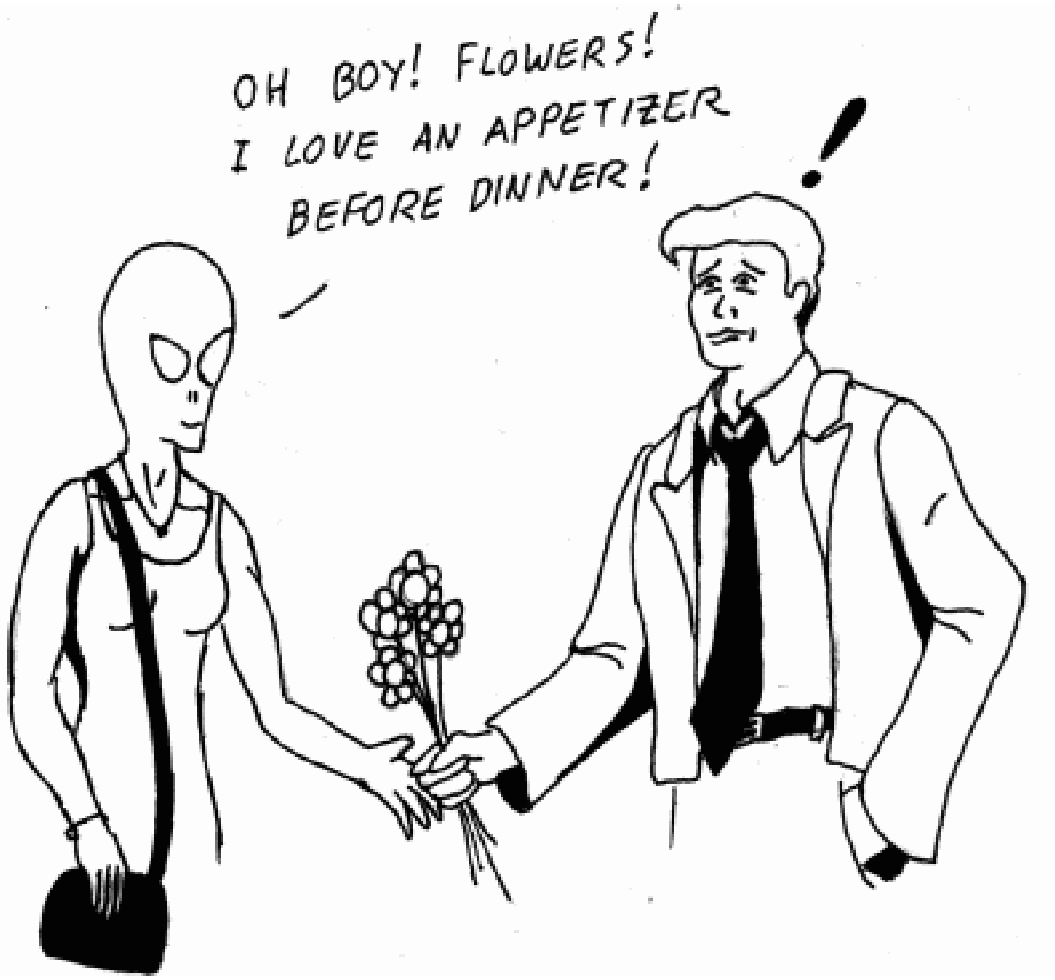
6. What happens when $\omega = \Omega$? Is this realistic?

7. Describe in your own words, as fully as you can, the analogy between the child on a swing and, say, the 2:1 orbital resonance between some hypothetical asteroid and Jupiter. (There are many equally valid ways of answering this question. Answer it in your own way.)

3 Intergalactic Dating

(**Quick question.**) Harold and Henrietta are dining tête-à-tête in the dining room of a spaceship moving forwards with speed $v = 0.9c$. Harold is facing the rear of the spaceship and Henrietta, opposite him, is facing the front. At one point in the conversation, they both agree to simultaneously raise their wine glasses. The raised glasses are separated by exactly 20 cm across the table, as perceived by Harold and Henrietta.

1. Draw a diagram of Harold and Henrietta, the glasses, and the spaceship.
2. To an observer outside the spaceship, looking into the window of the dining room as it whizzes by, what is the separation of the two glasses? Express your result in cm.
3. According to the same external observer, which glass is raised first? Harold's? Henrietta's? Or are they both raised at the same time?
4. Describe in your own words, succinctly, two **astrophysical** situations in which relativistic physics exerts a detectable influence over **orbital motions**.


OH BOY! FLOWERS!
I LOVE AN APPETIZER
BEFORE DINNER!
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